Shell-Fluid Coupled Simulation of Detonation-Driven Fracture and Fragmentation

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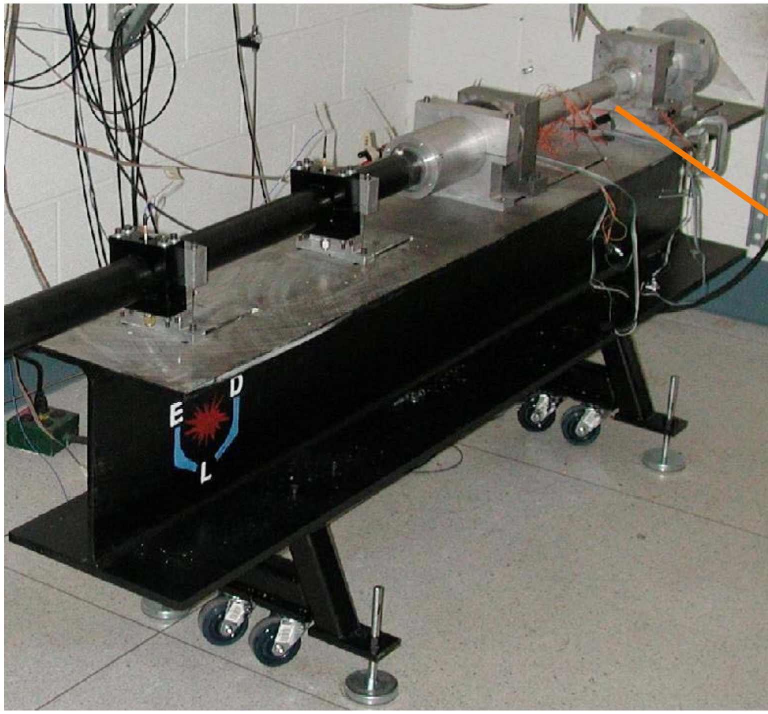
Al 6061-T6 Tube Fracture (J. Shepherd)



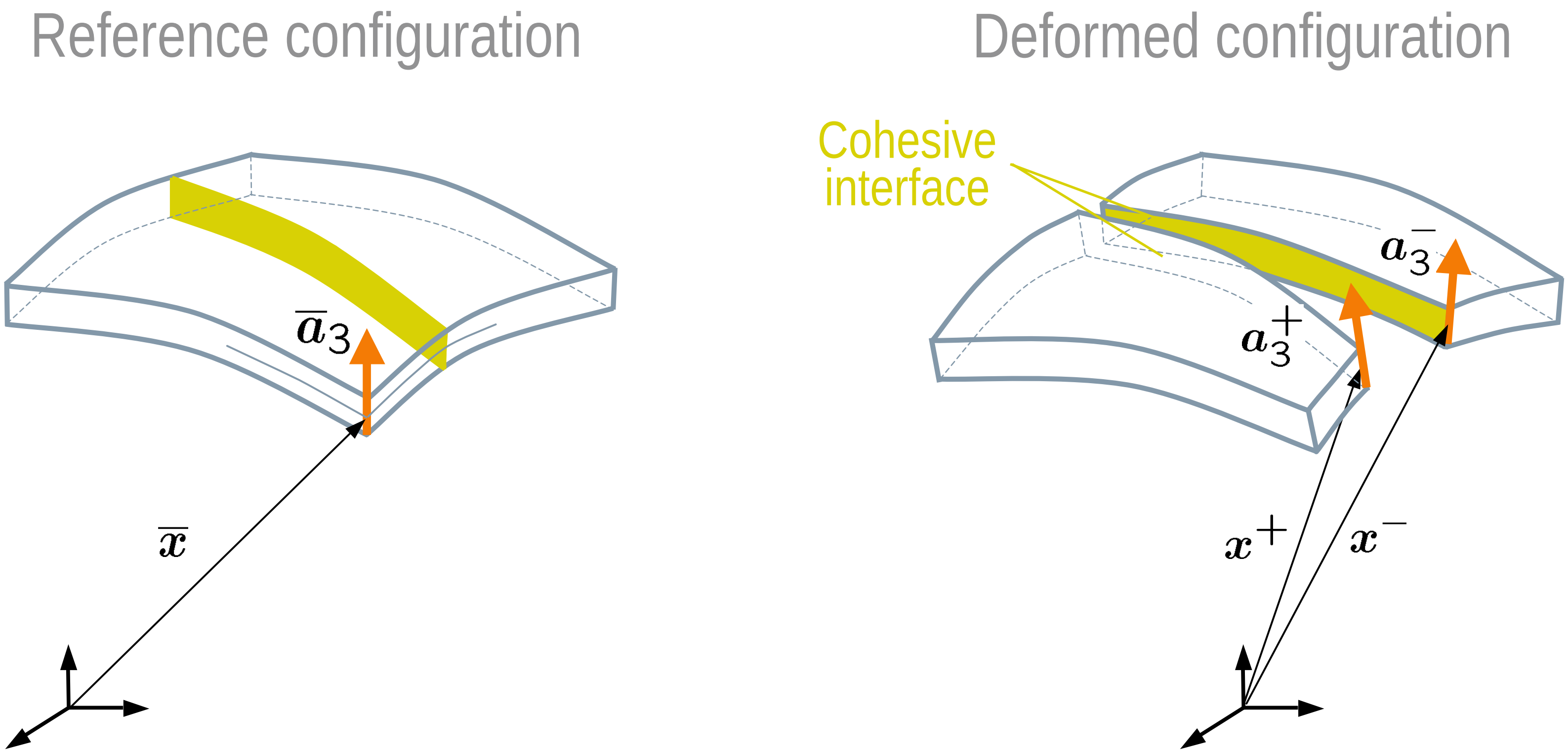
Experiments courtesy of J. Shepherd, Caltech

Fractured tubes

Ductile fracture

- Modeling and simulation challenges
  - Ductile mixed mode fracture with large deformations
  - Successive change of the mesh topology
  - Fluid-shell interaction under changing mesh topology

Fractured Thin-Shell Kinematics



$$\overline{r} = \overline{x}(\theta_1, \theta_2) + \theta^3 \overline{a}_3$$

$$r^{\pm} = x^{\pm}(\theta_1, \theta_2) + \theta^3 a_3^{\pm}$$

- Kirchhoff-Love assumption: Director a_3 is normal to the deformed middle surface

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Fractured Thin-Shell Equilibrium

- Shell and cohesive interface contribute to the internal virtual work

$$\delta\Pi^{int}_{Shell} + \delta\Pi^{int}_{Interface} - \delta\Pi^{ext} = 0$$

  - Shell internal virtual work consists of a membrane and bending term

$$\delta\Pi^{int}_{Shell} = \int (\boldsymbol{n}^{\alpha} \cdot \delta\boldsymbol{a}_{\alpha} + \boldsymbol{m}^{\alpha} \cdot \delta\boldsymbol{a}_{3,\alpha}) d\Omega$$

  - Cohesive internal virtual work consists of a tearing, shearing, and hinge term

$$\delta\Pi^{int}_{Interface} = \int (\boldsymbol{t} \cdot \delta[\![\boldsymbol{x}]\!] + \boldsymbol{s} \cdot \delta[\![\boldsymbol{a}_3]\!])\, d\Gamma_C$$

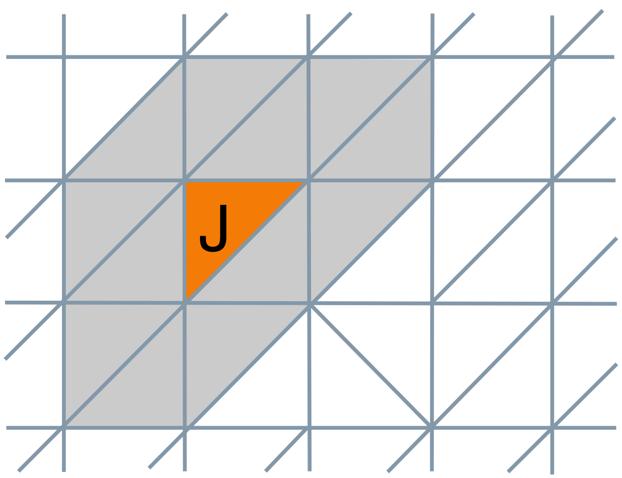
Subdivision FE-Discretization

- Away from crack flanks, conforming FE discretization requires smooth shape functions
  - On regular patches, quartic box-splines are used

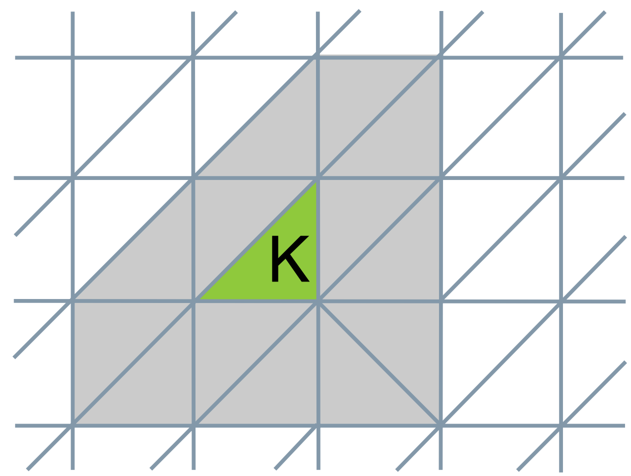
$$\overline{\boldsymbol{x}}_h(\xi,\eta) = \sum_{I=1}^{12} N^I(\xi,\eta)\overline{\boldsymbol{x}}_I$$

  - On irregular patches, subdivision schemes are used (here Loop's scheme)

Regular patch for element J



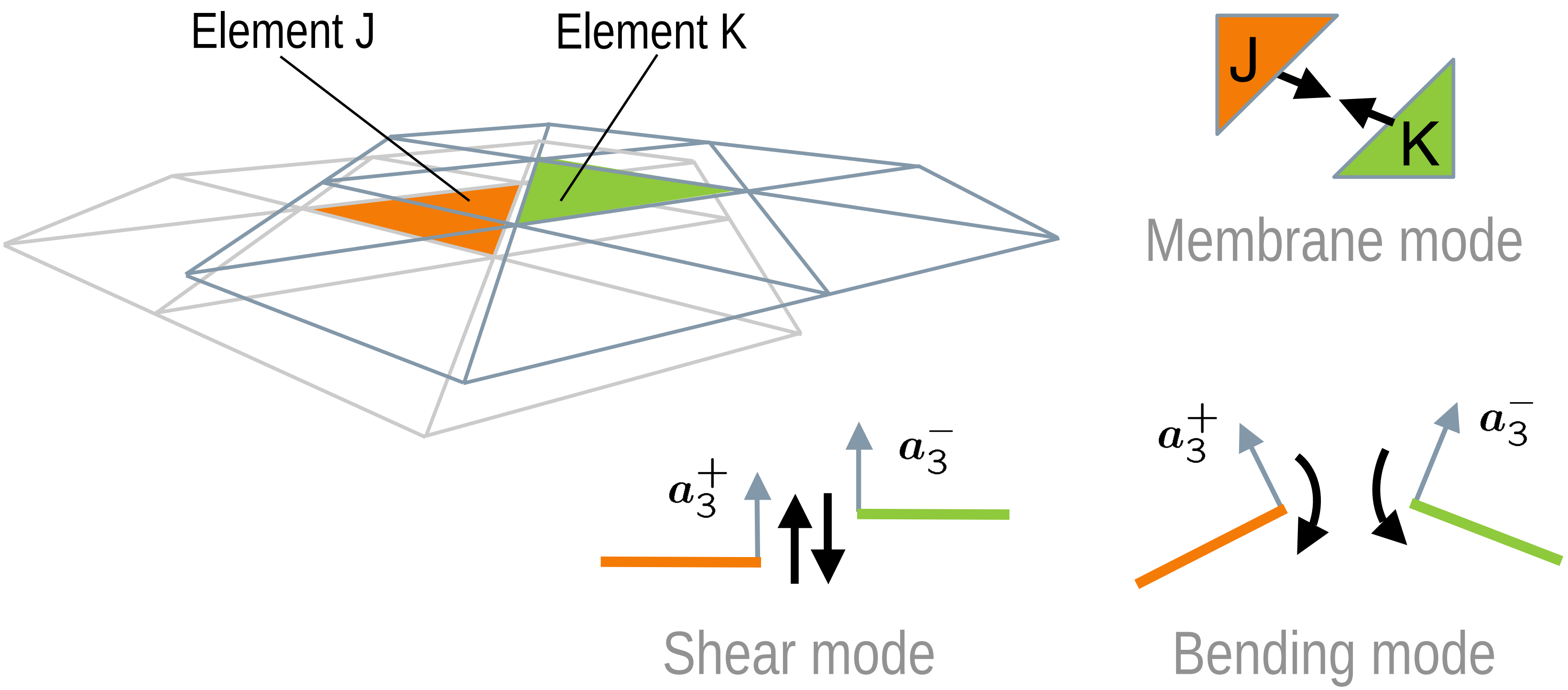
Irregular patch for element K



F. Cirak, M. Ortiz, P. Schröder, Int. J. Numer. Meth. Engrg. 47 (2000)

Discontinuous Shape Functions

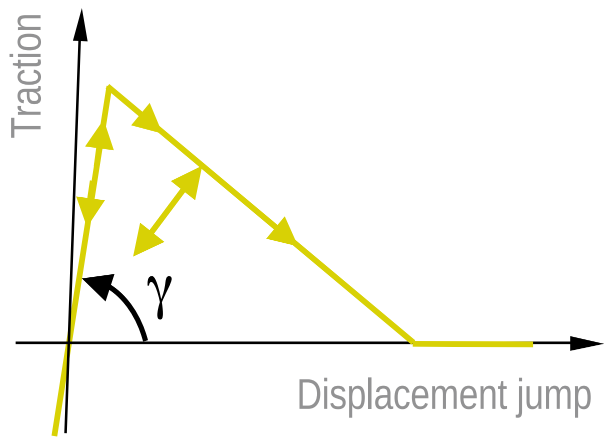
- Pre-fractured patches operate independently for interpolation purposes



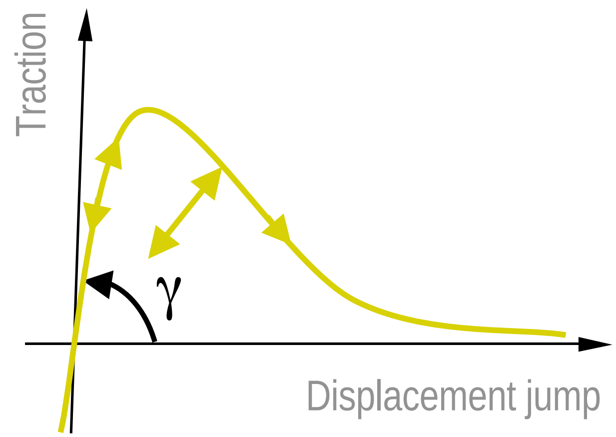
- Edge opening displacements and rotations activate cohesive tractions

Cohesive Interface Model

- Membrane, shear, and bending tractions are computed by numerical integration over shell thickness
  - At each quadrature point a conventional irreversible cohesive model is used



Linearly decreasing envelope

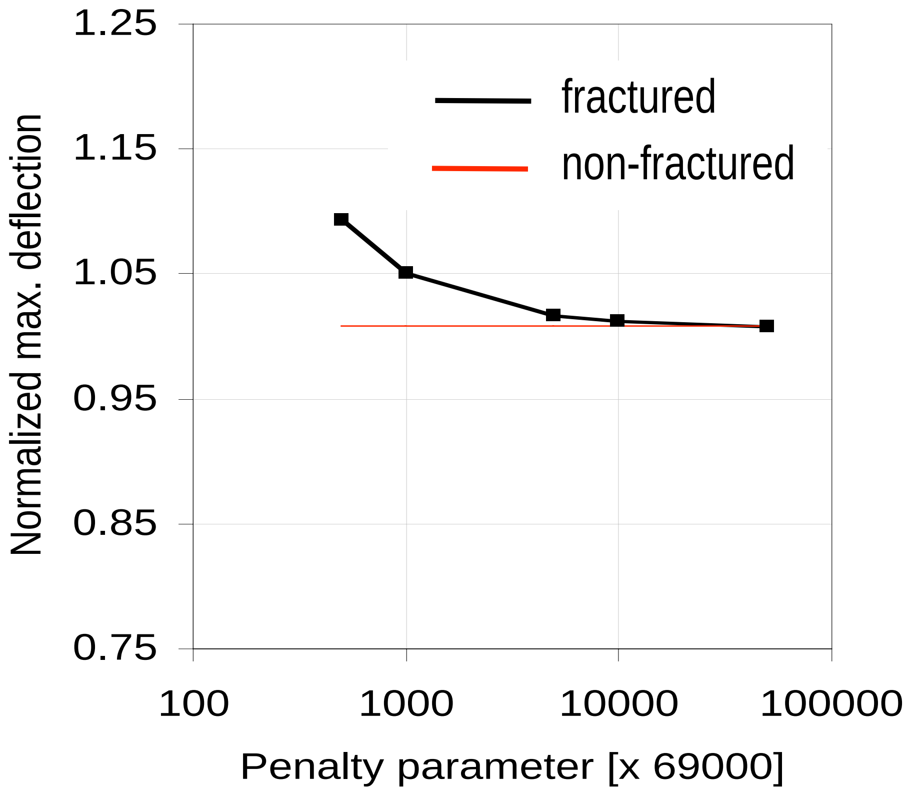


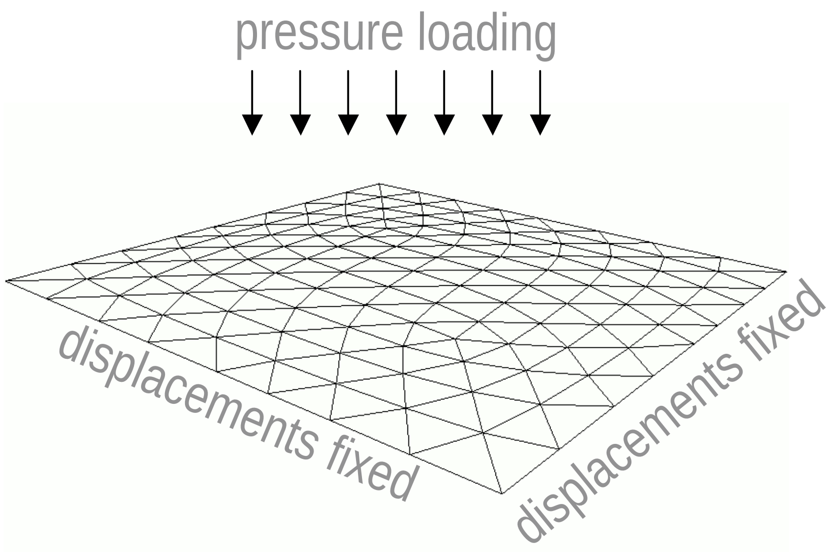
Smith-Ferrante envelope

- Conformity prior to crack initiation can be enforced

F. Cirak, M. Ortiz, A. Pandolfi, CMAME (2005)

Simply Supported Plate





Linear elastic material:	
Young's modulus	69000
Poisson's ratio	0.3

Geometry:	
Length	1.0
Thickness	0.1

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Fluid-Shell Coupling: Overview

- High speed flows interacting with thin-shells effectively require a coupled Eulerian-Lagrangian approach
  - In Eulerian formulations, mesh points are fixed
  - In Lagrangian formulations, mesh points follow the trajectories of material points

- Eulerian-Lagrangian coupling
  - Arbitrary Lagrangian Eulerian method
    - High accuracy, but algorithmically challenging for shells with large deformations
  - Interface tracking and Interface capturing schemes
    - Algorithmically very robust
    - Well established in Cartesian mesh based Eulerian fluid codes
    - Recently applied to fluid-solid coupling

Gas Dynamics

- Compressible inviscid fluid flow (Euler equations)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$ Mass conservation

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + \boldsymbol{I} p) = 0$$ Momentum conservation

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + p)\, \boldsymbol{v}] = 0$$ Energy conservation

- Specific total energy

$$E = \rho e + \frac{1}{2}\rho ||v||^2$$

- Equation of state for perfect gas

$$p = (\gamma - 1)\rho e$$

$\gamma -$ ratio of specific heats

Gas Dynamics - Discretization

- Euler equations in conservation law form

$$\boldsymbol{V}_{,t} + \nabla \cdot \boldsymbol{F} = 0$$

- Finite volume discretization on a Cartesian grid

$$\int_{\Omega} \boldsymbol{V}_{,t} dx + \int_{\Gamma} \boldsymbol{F} dn = 0$$

- Reduced to one dimensional problems along each coordinate axis using dimensional splitting

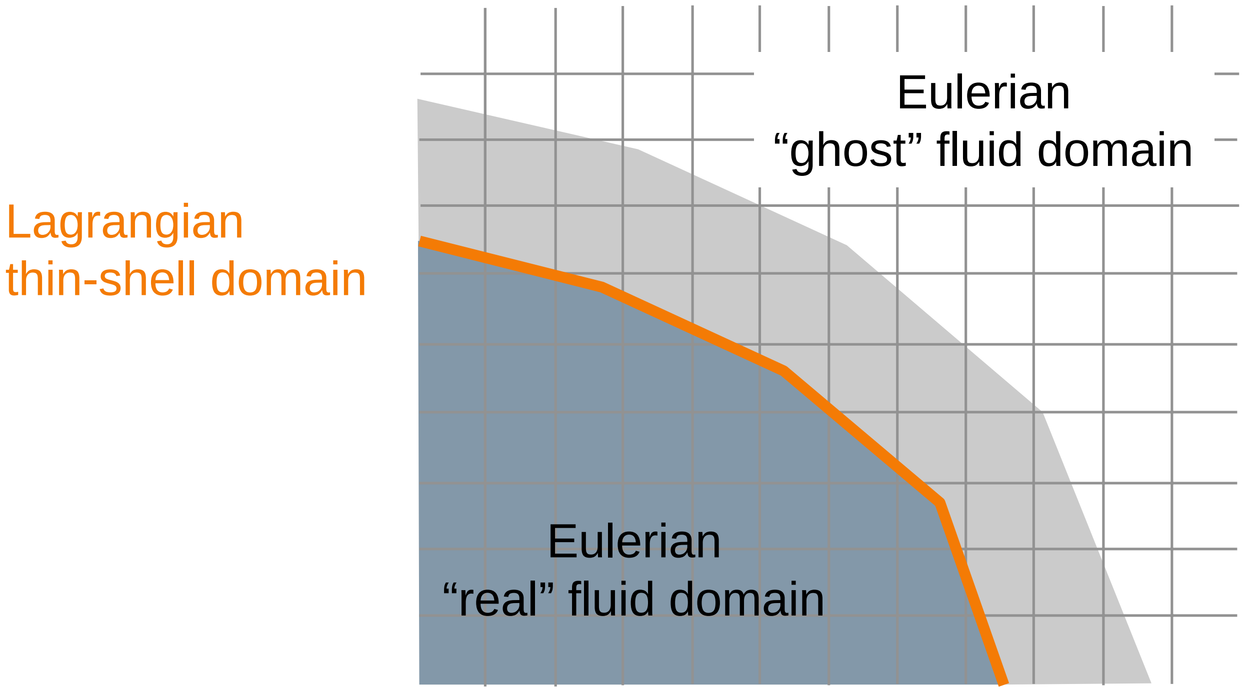
$$\frac{V_i^{n+1} - V_i^n}{\Delta t} h = \left(F^n_{i-\frac{1}{2}} - F^n_{i+\frac{1}{2}} \right) \qquad h - \text{ mesh size}$$

  - Fluxes are computed by solving local Riemann problems

- For additional features see amroc.sourceforge.net

Explicit Fluid-Shell Coupling -1-

- Thin-shell and fluid equations are integrated with an explicit time integration scheme



- Coupling is achieved by enforcing:
  - Continuity of normal velocity
  - Continuity of traction normal component
  - Unconstrained tangential slip

Explicit Fluid-Shell Coupling –2-

- Enforcing the interface conditions on the fluid grid through ghost-cells
  - Ghost cell values are extrapolated from the values at the shell-fluid interface
  - Normal velocity modifications in the ghost cells

$$\boldsymbol{v}_{Fluid} = [(2\tilde{\boldsymbol{v}}_{Shell} - \tilde{\boldsymbol{v}}_{Fluid}) \cdot \boldsymbol{n}]\, \boldsymbol{n} + (\tilde{\boldsymbol{v}}_{Fluid} \cdot \boldsymbol{t})\boldsymbol{t}$$

$\tilde{\boldsymbol{v}}_{Fluid}-$ extrapolated fluid velocity

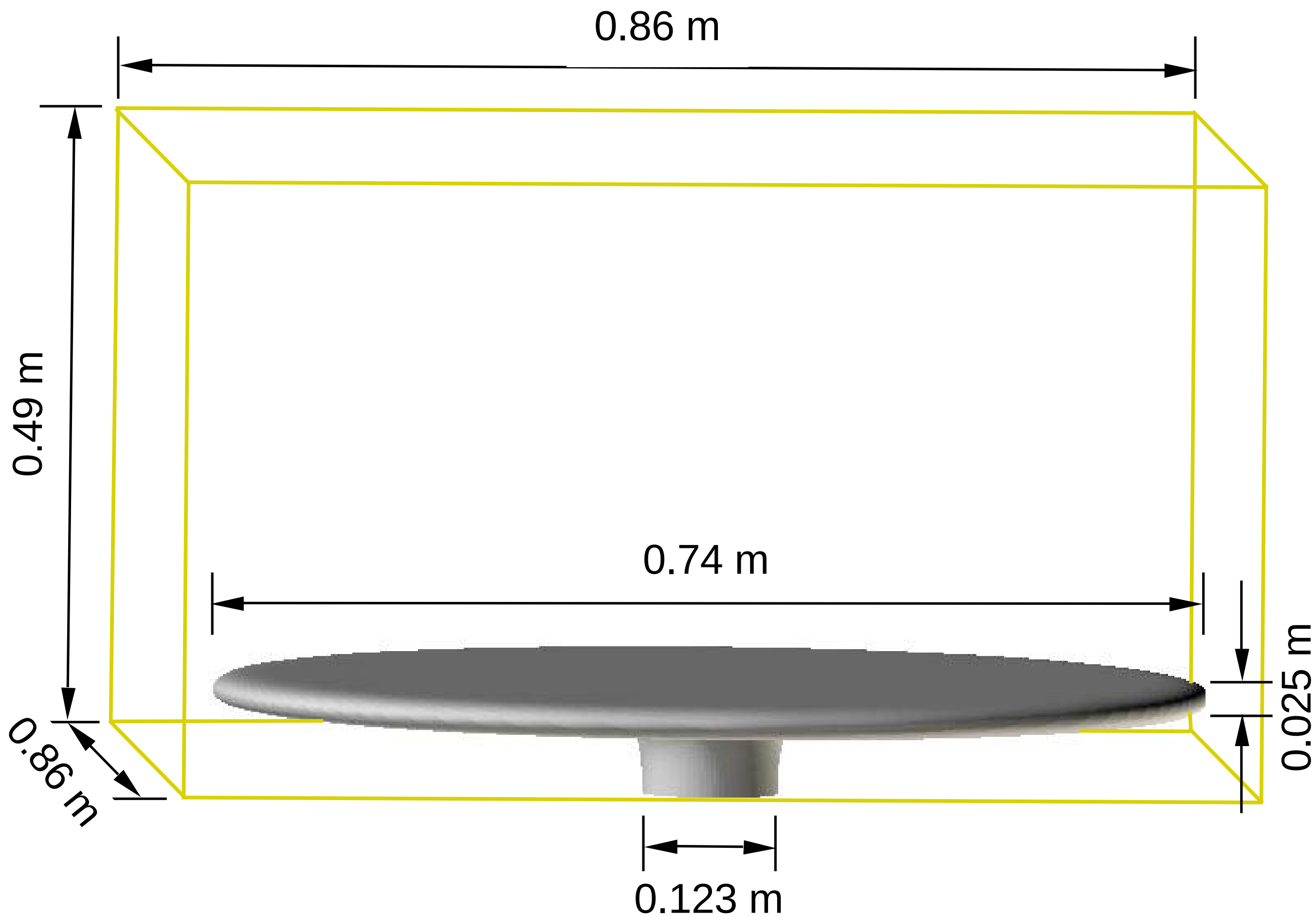
$\tilde{\boldsymbol{v}}_{Shell}-$ extrapolated shell velocity

$\boldsymbol{n}, \boldsymbol{t}-$ normal and tangent to the interface

  - Corresponds to reflecting the normal fluid velocity component in a moving local coordinate frame attached to the shell

- Enforcing the interface conditions on the shell
  - Interpolated pressures from the fluid mesh are applied as external traction boundary conditions to the shell

Airbag – Geometry and Discretization



Shell Mesh: 10176 elements

Fluid Mesh: 48x48x62 cells

F. Cirak, R. Radovitzky, C&S (2005)

Airbag – Limit Surface

total simulation
time
approx. 23 ms

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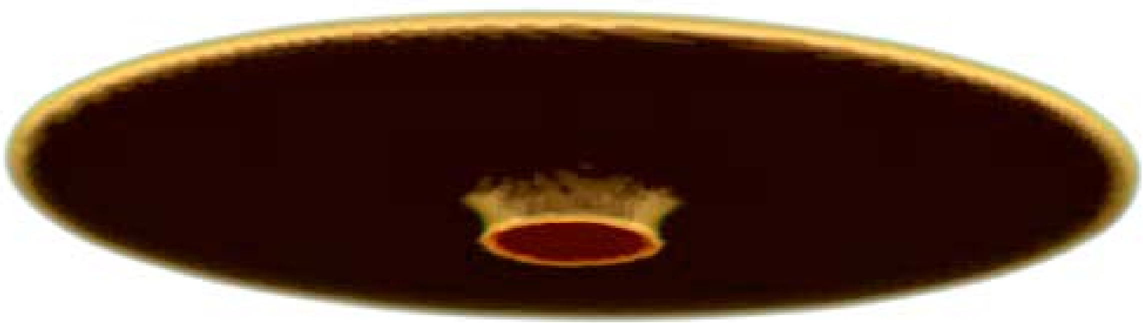
Airbag – Kinetic Energy



total simulation
time
approx. 23 ms

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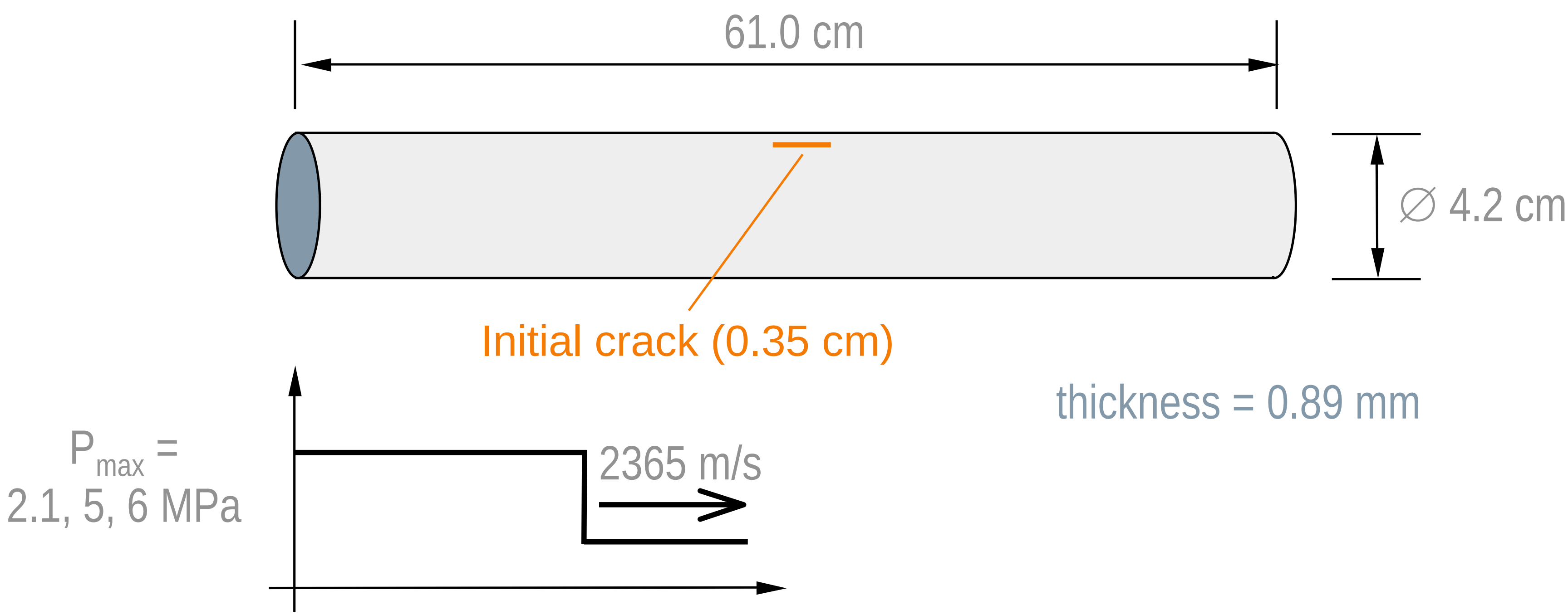
Fluid Induced Fracture



Material model for Al 6061-T6:
J_2 - plasticity with viscosity

Cohesive interface model:
Linearly decreasing envelope
with loading and unloading

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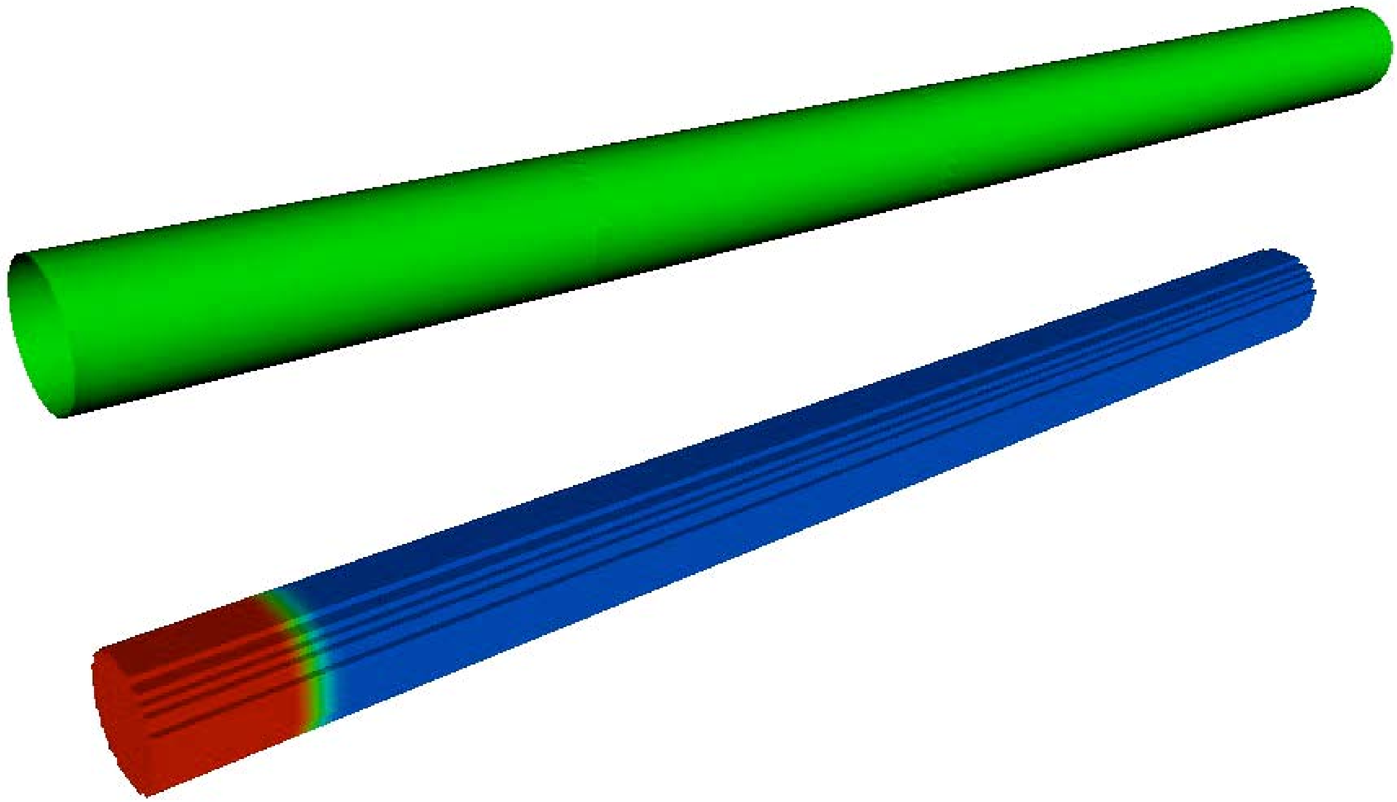
Coupled Simulation in Elastic Regime



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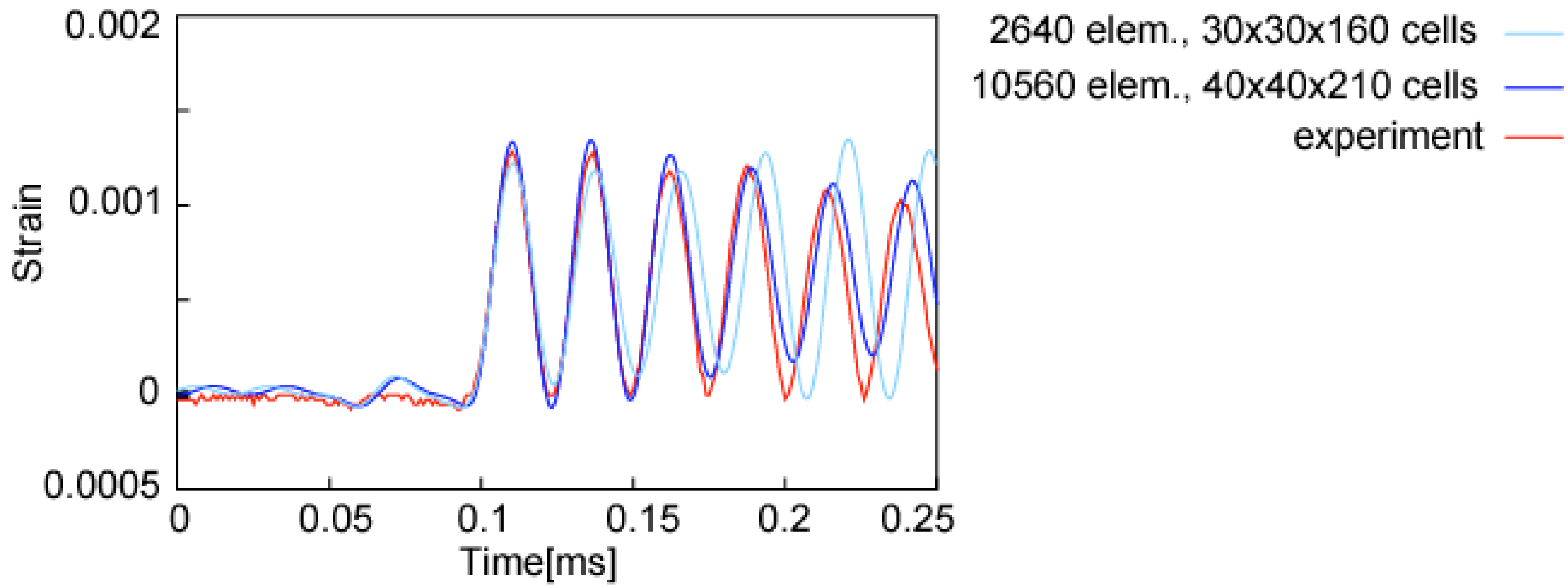
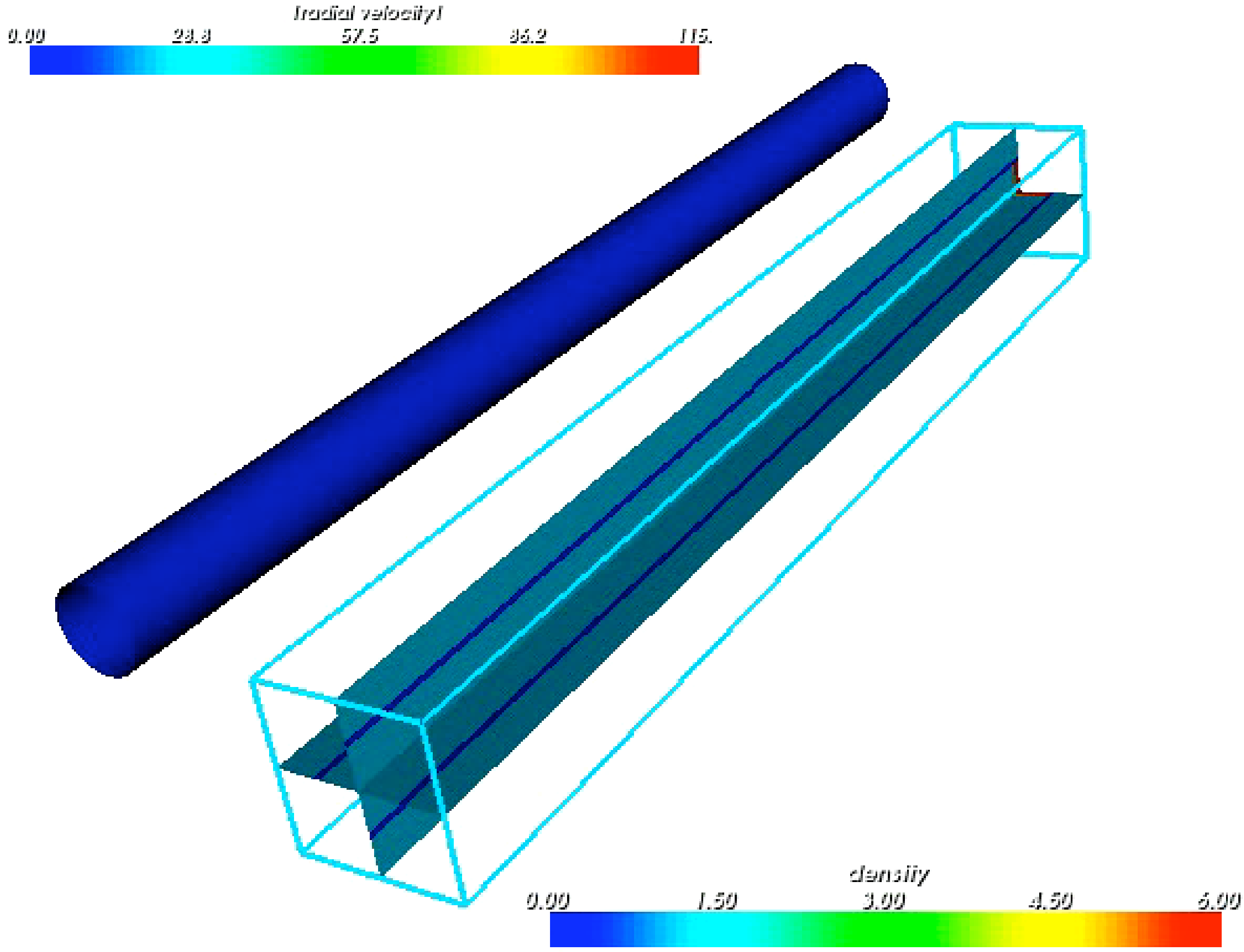
Coupled Simulation in Elastic Regime

- Circumferential strain at x=28.2cm



- The fluid grid initialized with normal shock conditions
- In contrast to the computation, the experiment performed with a detonation wave

Coupled Simulation with Fracture



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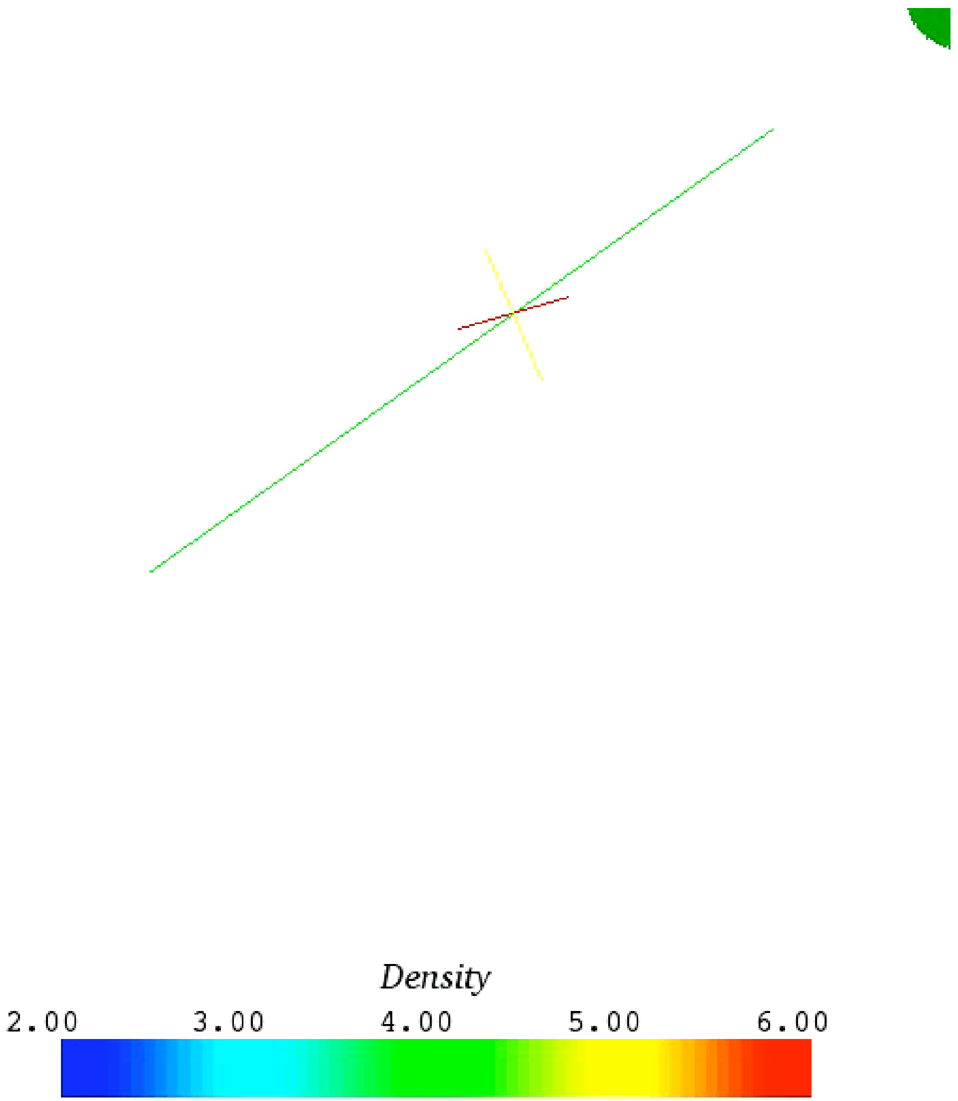
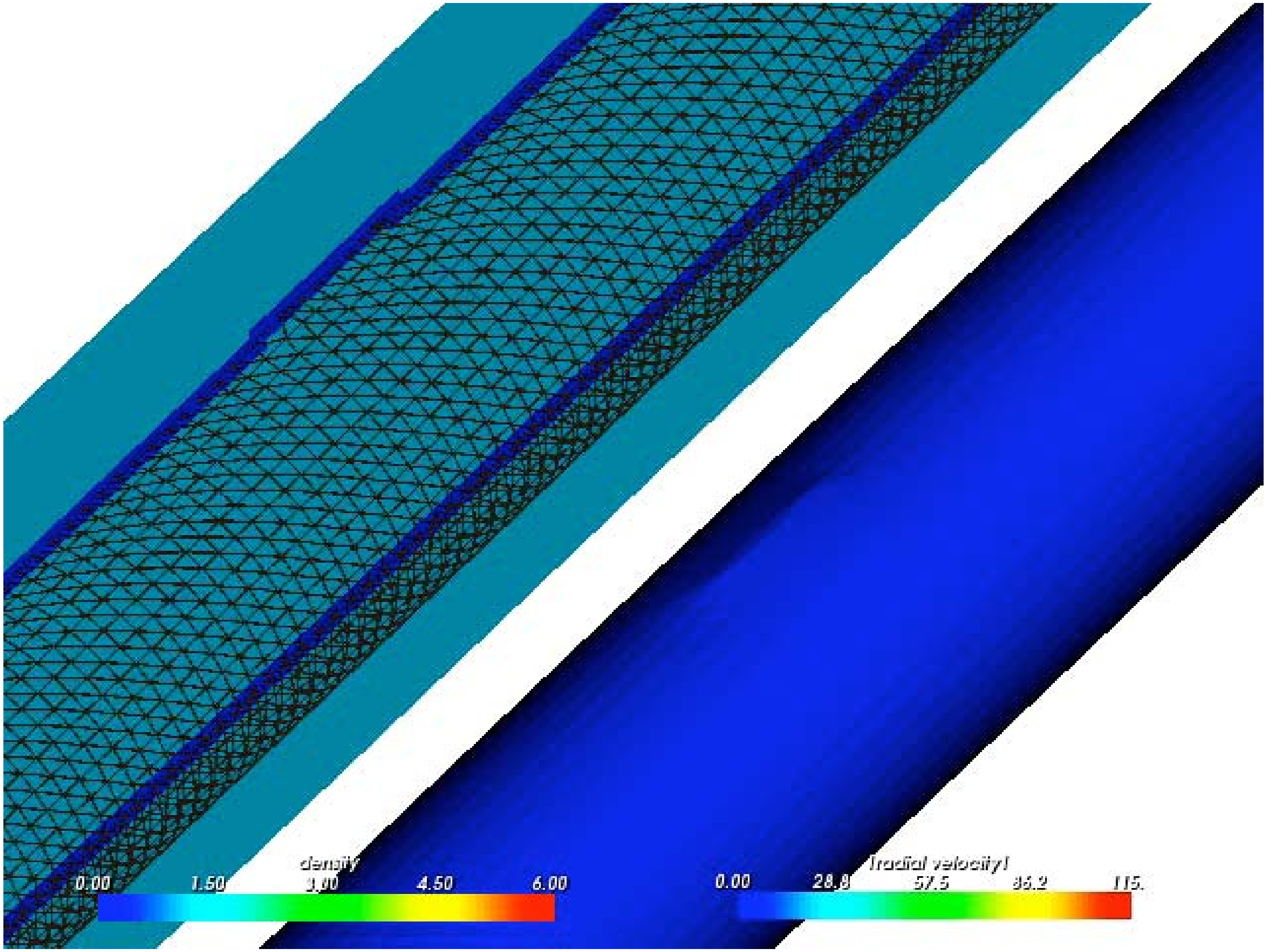
Coupled Simulation - Threshold



Coupled Simulation - Close-up



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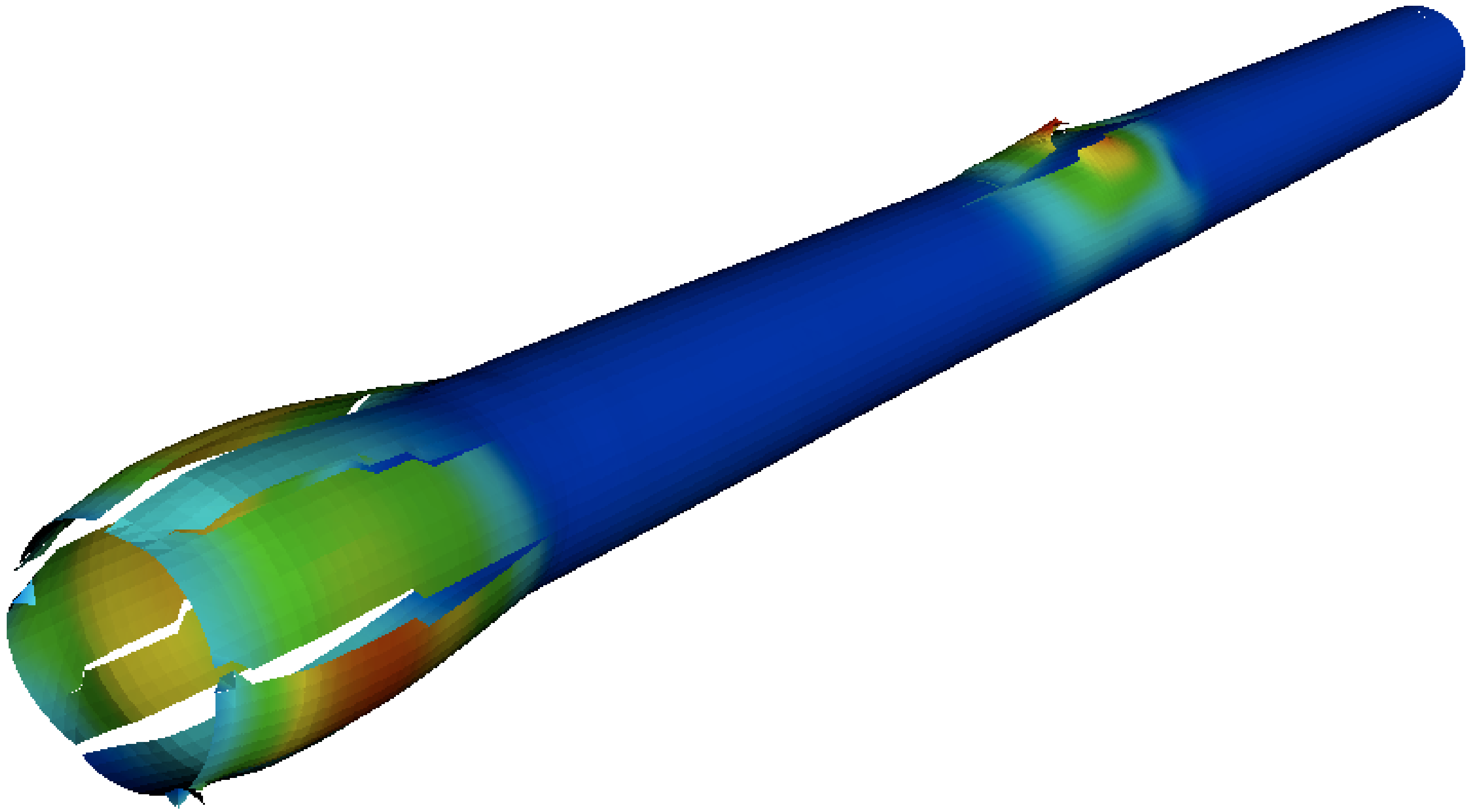
Coupled Simulation - P_{max} = 6.0MPa

- 20736 shell elements, 80x80x640 fluid cells



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Computational Specifics

- Computations performed on an Intel Xeon Cluster
  - 41 processors for the fluid
  - 29 processors for the shell
  - Total computing time ~8h

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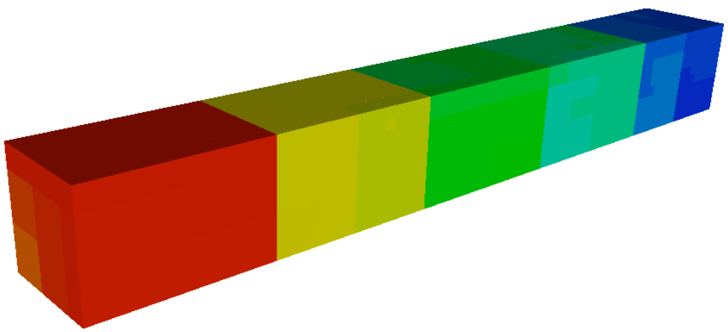
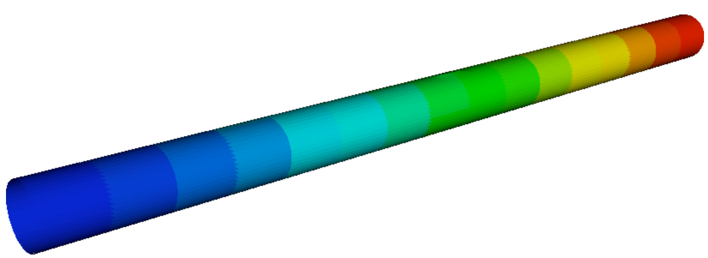
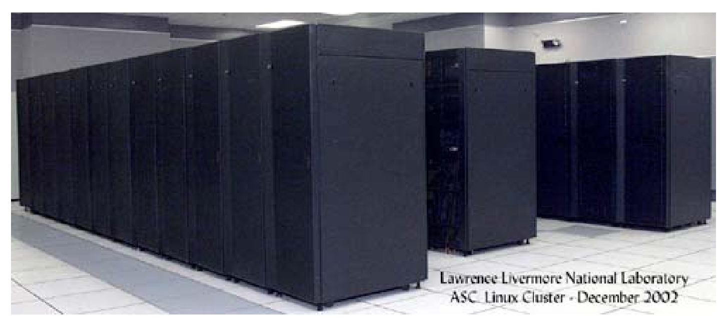


Partitioning of the fluid domain



Partitioning of the shell domain



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Conclusions

- The shell and the fluid, as well as their coupled interaction, are considered in full detail
- The proposed coupling approach is very robust and efficient
  - No remeshing
  - Algorithmic coupling with minor modifications of shell and fluid solvers
- Although first steps towards validation are encouraging, detonation tube experiments are far too challenging to simulate. Currently, a new set of shock tube experiments are done by J. Shepherd at Caltech.