Shell-Fluid Coupled Simulation of Detonation-Driven Fracture and Fragmentation

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Al 6061-T6 Tube Fracture (J. Shepherd)

Experiments courtesy of J. Shepherd, Caltech

Modeling and simulation challenges
- Ductile mixed mode fracture with large deformations
- Successive change of the mesh topology
- Fluid-shell interaction under changing mesh topology

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Fractured Thin-Shell Kinematics

**Reference configuration**

\[ \overline{r} = \overline{x}(\theta_1, \theta_2) + \theta^3 \overline{a}_3 \]

**Deformed configuration**

\[ r^\pm = x^\pm(\theta_1, \theta_2) + \theta^3 a^\pm_3 \]

- Kirchhoff-Love assumption: Director \( a_3 \) is normal to the deformed middle surface
Fractured Thin-Shell Equilibrium

- Shell and cohesive interface contribute to the internal virtual work

\[ \delta \Pi_{\text{Shell}}^{\text{int}} + \delta \Pi_{\text{Interface}}^{\text{int}} - \delta \Pi_{\text{ext}} = 0 \]

- Shell internal virtual work consists of a membrane and bending term

\[ \delta \Pi_{\text{Shell}}^{\text{int}} = \int (n^\alpha \cdot \delta a_\alpha + m^\alpha \cdot \delta a_{3,\alpha}) d\Omega \]

- Cohesive internal virtual work consists of a tearing, shearing, and hinge term

\[ \delta \Pi_{\text{Interface}}^{\text{int}} = \int (t \cdot \delta [x] + s \cdot \delta [a_3]) d\Gamma_C \]
Subdivision FE-Discretization

- Away from crack flanks, conforming FE discretization requires smooth shape functions
  - On regular patches, quartic box-splines are used
    \[ \bar{x}_h(\xi, \eta) = \sum_{I=1}^{12} N^I(\xi, \eta) \bar{x}_I \]
  - On irregular patches, subdivision schemes are used (here Loop's scheme)

Discontinuous Shape Functions

- Pre-fractured patches operate independently for interpolation purposes
- Edge opening displacements and rotations activate cohesive tractions
Cohesive Interface Model

- Membrane, shear, and bending tractions are computed by numerical integration over shell thickness
  - At each quadrature point a conventional irreversible cohesive model is used

- Conformity prior to crack initiation can be enforced

F. Cirak, M. Ortiz, A. Pandolfi, CMAME (2005)
Simply Supported Plate

Linear elastic material:
Young’s modulus       69000
Poisson’s ratio       0.3

Geometry:
Length       1.0
Thickness    0.1

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Fluid-Shell Coupling: Overview

- High speed flows interacting with thin-shells effectively require a coupled Eulerian-Lagrangian approach
  - In Eulerian formulations, mesh points are fixed
  - In Lagrangian formulations, mesh points follow the trajectories of material points

- Eulerian-Lagrangian coupling
  - Arbitrary Lagrangian Eulerian method
    - High accuracy, but algorithmically challenging for shells with large deformations
  - Interface tracking and Interface capturing schemes
    - Algorithmically very robust
    - Well established in Cartesian mesh based Eulerian fluid codes
    - Recently applied to fluid-solid coupling
Gas Dynamics

- Compressible inviscid fluid flow (Euler equations)

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \text{Mass conservation}
\]

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + I \rho) = 0 \quad \text{Momentum conservation}
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot [(E + p) \mathbf{v}] = 0 \quad \text{Energy conservation}
\]

- Specific total energy

\[ E = \rho e + \frac{1}{2} \rho ||v||^2 \]

- Equation of state for perfect gas

\[ p = (\gamma - 1) \rho e \quad \gamma \text{— ratio of specific heats} \]
Gas Dynamics - Discretization

- Euler equations in conservation law form
  \[ V_{i,t} + \nabla \cdot F = 0 \]

- Finite volume discretization on a Cartesian grid
  \[ \int_{\Omega} V_{i,t} dx + \int_{\Gamma} F dn = 0 \]

- Reduced to one dimensional problems along each coordinate axis using dimensional splitting
  \[ \frac{V_{i}^{n+1} - V_{i}^{n}}{\Delta t} h = \left( F_{i-\frac{1}{2}}^{n} - F_{i+\frac{1}{2}}^{n} \right) \quad h - mesh size \]

  - Fluxes are computed by solving local Riemann problems

- For additional features see amroc.sourceforge.net
Thin-shell and fluid equations are integrated with an explicit time integration scheme. Coupling is achieved by enforcing:

- Continuity of normal velocity
- Continuity of traction normal component
- Unconstrained tangential slip

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Explicit Fluid-Shell Coupling –2–

- **Enforcing the interface conditions on the fluid grid through ghost-cells**
  - Ghost cell values are extrapolated from the values at the shell-fluid interface
  - Normal velocity modifications in the ghost cells
    
    \[
    \mathbf{v}_{\text{Fluid}} = \left[(2\mathbf{\tilde{v}}_{\text{Shell}} - \mathbf{\tilde{v}}_{\text{Fluid}}) \cdot \mathbf{n}\right] \mathbf{n} + (\mathbf{\tilde{v}}_{\text{Fluid}} \cdot \mathbf{t}) \mathbf{t}
    \]
    
    - \(\mathbf{\tilde{v}}_{\text{Fluid}}\) — extrapolated fluid velocity
    - \(\mathbf{\tilde{v}}_{\text{Shell}}\) — extrapolated shell velocity
    - \(\mathbf{n}, \mathbf{t}\) — normal and tangent to the interface
  - Corresponds to reflecting the normal fluid velocity component in a moving local coordinate frame attached to the shell

- **Enforcing the interface conditions on the shell**
  - Interpolated pressures from the fluid mesh are applied as external traction boundary conditions to the shell
Airbag – Geometry and Discretization

Shell Mesh: 10176 elements

Fluid Mesh: 48x48x62 cells

F. Cirak, R. Radovitzky, C&S (2005)
Airbag – Limit Surface

total simulation time approx. 23 ms

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Airbag – Kinetic Energy

total simulation time
approx. 23 ms

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**Fluid Induced Fracture**

- Initial crack (0.35 cm)
- Thickness = 0.89 mm
- Diameter = 4.2 cm
- Length = 61.0 cm

**Material model for Al 6061-T6:**
- $J_2$ - plasticity with viscosity

**Cohesive interface model:**
- Linearly decreasing envelope with loading and unloading

$P_{\text{max}} = 2.1, 5, 6 \text{ MPa}$

2365 m/s
Coupled Simulation in Elastic Regime

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Coupled Simulation in Elastic Regime

- Circumferential strain at $x=28.2\text{cm}$

![Graph showing strain over time with different grid resolutions and experimental data.](image)

- The fluid grid initialized with normal shock conditions
- In contrast to the computation, the experiment performed with a detonation wave
Coupled Simulation with Fracture
Coupled Simulation - Threshold
Coupled Simulation - Close-up
Coupled Simulation - $P_{\text{max}} = 6.0\text{MPa}$

- 20736 shell elements, 80x80x640 fluid cells
Computational Specifics

- Computations performed on an Intel Xeon Cluster
  - 41 processors for the fluid
  - 29 processors for the shell
  - Total computing time ~8h

Partitioning of the fluid domain

Partitioning of the shell domain

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Conclusions

- The shell and the fluid, as well as their coupled interaction, are considered in full detail.

- The proposed coupling approach is very robust and efficient:
  - No remeshing
  - Algorithmic coupling with minor modifications of shell and fluid solvers

- Although first steps towards validation are encouraging, detonation tube experiments are far too challenging to simulate. Currently, a new set of shock tube experiments are done by J. Shepherd at Caltech.